# Analysis of the Maximum and the Minimum of the Spectral Transmittance According to the Coherence Condition for Weakly Absorbing Thin Films.

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*In this work the maximum* $T_{sam}^{max}$ *and the minimum* $T_{sam}^{min}$ *of the spectral transmittance of weakly absorbing thin films deposited on non-absorbing substrate is analyzed as a function of refractive indices of both substrate and film according to the coherence condition. Also the ratio of* $T_{sam.}/T_w$ *(where* $T_w$ *is the transmittance of the uncoated substrate) is obtained for both homogeneous and inhomogeneous films. The results of the analysis enabled us to interpret the increment of the ratio* $(T_{sam.} / T_w)$ *over unity, distinguishing the inhomogeneous and homogenous films and obtaining the refractive index of the film.*

## Introduction

The effect of sample type on the spectrophotometric results occurs by two ways, interference within the film as well as the substrate which depends upon the coherence condition and the interaction between sample and photometer.

It is well known that the methods devoted for determining the optical constants of metal or dielectric thin films, [1], were based on analysis of the spectral scans of transmittance and reflectance. One of them based on the determination of the spectral transmittance of the coated $T_{sam.}$ relative to the uncoated substrate $T_w$ which is defined as the observed transmittance where ($T_{obs} = T_{sam}/ T_w$) ,[2 ]. It was noticed experimentally that $T_{obs}$ sometimes exceeds unity.

In this work different cases of coherency are considered. Also analysis of the spectral transmittance is driven.

**Theory:**

Figure (1) shows a practical situation where a parallel and monochromatic beam is incident on homogeneous, absorbing or non-absorbing, and isotropic film of thickness $d_1$ with a plane parallel surfaces. The substrate is being transparent with thickness $d_2$.

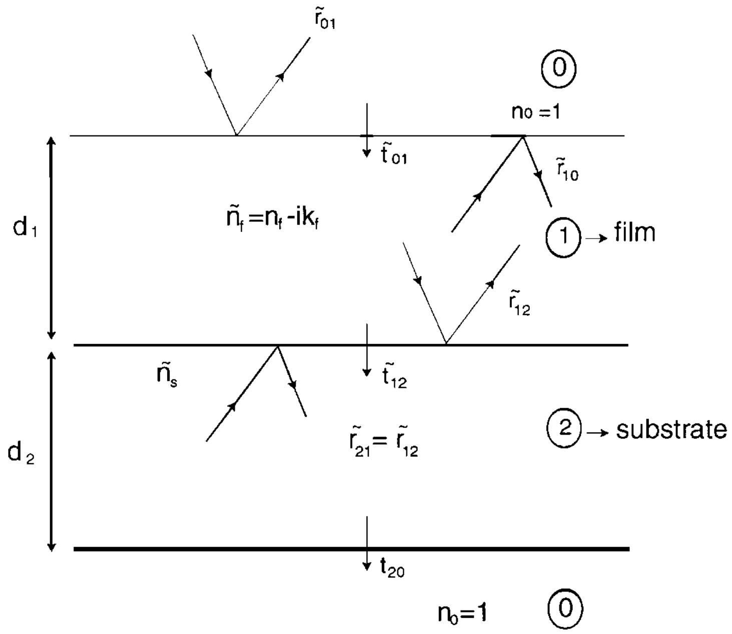


**Fig. (2)** The optical constants used in the calculation of transmittance of a thin film on a nonabsorbing substrate.

Generally the transmittance $T_{sam}$ can be expressed in terms of complex transmission ($\hat{t}$) as follows, [3],

$$T_{sam} = /\hat{t}_{sam}/^2 \tag{1}$$

where

$$\hat{t}_{sam} = \frac{\hat{t}_{01}\hat{t}_{12}\hat{t}_{20}e^{-i(\hat{\beta}_1+\hat{\beta}_2)}}{(1+\hat{r}_{01}\hat{r}_{12}e^{-2i\hat{\beta}_1}) + (\hat{r}_{12}+\hat{r}_{01}e^{-2i\hat{\beta}_1})\hat{r}_{20}e^{-2i\hat{\beta}_2}}$$

Where the $\hat{r}_{ij}$ , $\hat{t}_{ij}$ are the Fresnel coefficients for a light beam going from medium i with complex refractive index $\hat{n}_i$ to medium j with refractive index $\hat{n}_j$, $\hat{\beta}_1$ and $\hat{\beta}_2$ are the complex phase factors for the two traversal media respectively.

Also $$\hat{r}_{ij} = \frac{\hat{n}_i - \hat{n}_j}{\hat{n}_i + \hat{n}_j}, \qquad \hat{t}_{ij} = \frac{2\hat{n}_i}{\hat{n}_i + \hat{n}_j},$$

$$\hat{\beta}_1 = \frac{2\pi}{\lambda}\hat{n}_l d_l \qquad \hat{\beta}_2 = \frac{2\pi}{\lambda}\hat{n}_2 d_2 \tag{2}$$

where $\lambda$ is the wavelength, For semi-finite substrate $\hat{t}$ is given by, [4],

$$\hat{t} = \hat{t}_f \frac{t_{20}\, e^{-i\beta_2}}{1-\hat{r}_f r_{20} e^{-2i\beta_2}} = t_f t_s \tag{3}$$

where $\hat{t}_f$ is refered to the complete transmitted amplitude of the absorbing film and $\hat{t}_s$ is that of the substrate

$$\hat{t}_f = \hat{t}_{01}\hat{t}_{12}e^{-i\hat{\beta}_2} \times \frac{1}{1-\hat{r}_{10}\hat{r}_{12}e^{-2i\hat{\beta}_2}} \tag{4}$$

and $$\hat{t}_s = \frac{\hat{t}_{20}\, e^{-i\beta_2}}{1-\hat{r}'_f \hat{r}_{20} e^{-2i\beta_2}} \tag{5}$$

where $r'_f$ represents the reflection amplitude of the film when the light incident from the substrate side and given by

$$\hat{r}_f' = \frac{\hat{r}_{21} + \hat{r}_{10}\, e^{-2i\hat{\beta}_1}}{1 - \hat{r}_{10}\hat{r}_{12}\, e^{-2i\hat{\beta}_1}} \tag{6}$$

The interference occurs when the band width of the spectrophotometer $\Delta\lambda$, the thickness and optical constants of the film (or substrate) verifying the coherence condition.

$$\Delta\lambda.2\pi.n.d / \lambda^2 << 1 \tag{7}$$

In this case the transmittance through the film (or substrate) obtained by adding the coherent amplitudes of the multireflections in the film or substrate are denoted by $T_{f,coh}$ ($T_{s,coh}$) which can be given by equations (3,6). In the case for which the coherence condition is not valid the multiple reflections will be added incoherently and the obtained transmittance are denoted by $T_{f,incoh}$ and $T_{s,incoh}$ . Then to reveal the effect of interference and inhomogenity on the transmittance of the sample, we have to consider the cases for which $T_{f,coh}$ and $T_{s,coh}$ for homogenous and inhomogenous films. Also the influence of using $T_{s,incoh}$ is considered.

It is worth noting that all the derivations have been carried out assuming weakly absorbing film for which $K_2 << |n_2 - n_1|, |n_2 - n_3|$.

**Case 1:**

**A single non-absorbing film on a non-absorbing substrate.**
**$T_{f,coh}$ and $T_{s,coh}$ for homogenous films.**

In this case $T_{f,coh}$ can be given by equation (4) it is found that $T^{max}_{f,coh} = 4n_s/(n_s + 1)^2$. Also from equations (5) and (6 ) we have

$$T_{s,coh} = \frac{(1 - R_{20})}{(1 - 2|r_f'||r_{20}| + |r_f'|^2 R_{20})} \tag{8}$$

As the maximum value of $T_{s,coh}$ occurs at $\beta_1 = 2m\pi$ where m = 0,1,2 ,.. for which $|r_f'|^2 = R_{20}$, then $T^{max}_{s,coh}$ can be written in the following formula

$$T^{max}_{s,coh} = \frac{(1 - R_{20})}{(1 - R_{20})^2} = \frac{(n_S + 1)^2}{4n_S} \tag{9}$$

So it is easy to show that the maximum value of the sample transmittance $T_{sam}^{max}$ in this case is equal to unity. On the other hand, the maximum value of $T_{W,coh}$ for the uncoated substrate is equal to unity and the minimum value is given by,[6],

$$T_{w,coh}^{min} = \frac{4n_w^2}{(n_w^2+1)^2} \tag{10}$$

The minimum value of the transmittance can be obtained using $T_{f,coh}^{min}$ from equation (4) and $T_{s,coh}^{min}$ using equation (5).Then $T_{sam}^{min}$ can be written as

$$T_{sam}^{min} = \frac{4n_1^2}{(n_1^2+1)^2} \tag{11}$$

Then using equation (10) we have.

$$T_{obs}^{min} = \frac{n_1^2(n_w^2+1)^2}{n_w^2(n_1^2+1)^2} \tag{12}$$

It is clear that $T_{obs}^{max}$ is equal to unity in this case , therefore $T_{obs}^{max}$ >1 is not probable in such a case. From equation (11) it is clear that $T_{sam}^{min}$ independent on $n_s$ and the variations of $T_{sam}^{min}$ , $T_{obs}^{min}$ and the quantity ($T_{obs}^{max}$ - $T_{obs}^{min}$) with $n_f$ are considerable and illustrated in table (1) .

**Case 2 :**

**Inhomogeous film, $T_{f,coh}$ and $T_{s,incoh}$ .**

In case of inhomogenous film on a transparent substrate. According to, [8] ,it is assumed that the refractive index of a film at the film/air interface $n_f$ and at the film/substrate interface $n_b$ are different, then $T_f$ can be detemined for such case.

$$T_f = \frac{16 n_f n_b n_s A}{C_1^2 + C_2^2 A^2 + 2C_1 C_2 A\cos(4\pi n_m d/\lambda)} \tag{13}$$

Where $C_1 = (n_f + n_o)(n_1 + n_b)$, $C_2 = (n_f - n_o)(n_1 - n_b)$, $A = \exp(-\alpha d_1)$ $\alpha$ denotes the absorption coefficient of the film and $n_m$ is the mean refractive index of the film having thickness $d_1$.

Putting A=1 in the case of weakly aborbing films, then from equation (4), we can write

$$T_{f,coh}^{\max} = \frac{4n_f n_b n_s}{(n_s n_f + n_b)^2} \quad (14)$$

From equation (14), it is clear that $T_{f,coh}^{\max}$ is dependent on $\Delta n = (n_f - n_b)$. Also according to equation (5) a formula for $T_{s,\,incoh}^{\max}$ in this case is given by

$$T_{s,\,incoh}^{\max} = \frac{(n_s n_f + n_b)^2}{(n_s^2 n_f n_b + n_s^2 n_f^2 + n_b^2 + n_f n_b)} \quad (15)$$

Then

$$T_{sam}^{\max} = \frac{4n_f n_b n_s}{(n_S^2 n_f n_b + n_f^2 n_S^2 + n_b^2 + n_f n_b)} \quad (16)$$

Since $T_{s,incoh}$ in this case then we have $T_{w,incoh}$ which detemined by,[9],

$$T_{w,\,incoh}^{\max} = \frac{2n_w^2}{n_w^2 + 1} \quad (17)$$

$$T_{obs}^{max} = \frac{2n_f n_b (n_S^2 + 1)}{(n_f n_b n_S^2 + n_f^2 n_S^2 + n_b^2 + n_f n_b)} \quad (18)$$

To obtain $T_{sam}^{\min}$ we must have $T_{f,coh}^{\min}$ , $T_{s,incoh}^{\min}$ . So from equation (4) we have

$$T_{f,coh}^{\min} = \frac{4n_f n_b n_S}{(n_f^2 n_b^2 + 2n_f n_b n_S + n_S^2 n_0^2)} \quad (19)$$

Then from equations (5) and (6) we have

$$T_{s,incoh}^{\min} = \frac{(n_f n_b + n_S)^2}{(n_f n_b + n_S^2)(n_f n_b + 1)} \quad (20)$$

$$T_{sam}^{\min} = \frac{4n_f n_b n_S}{(n_f^2 n_b^2 + n_f n_b n_S^2 + n_f n_b + n_S^2)} \quad (21)$$

Then using equation (12) considering $n_S = n_{w,}$ then we have;

$$T^{\min}_{\text{obs}} = \frac{2n_f n_b (n_S^2 + 1)}{(n_f^2 n_b^2 + n_f n_b (n_S^2 + 1) + n_S^2)} \qquad (22)$$

The calculations of $T^{max}_{obs}$ and $T^{\min}_{\text{obs}}$ versus Δn for different values of $n_m$ show that $T^{max}_{obs}$ is dependent on Δn and $n_m$ only. The results of calculations is illustrated in Fig. (2).Also it has been found that the quantity ( $T^{max}_{obs}$ - $T^{\min}_{\text{obs}}$ ) markedly varies with $n_m$ and slightly with Δn as illustrated in Tables (2).

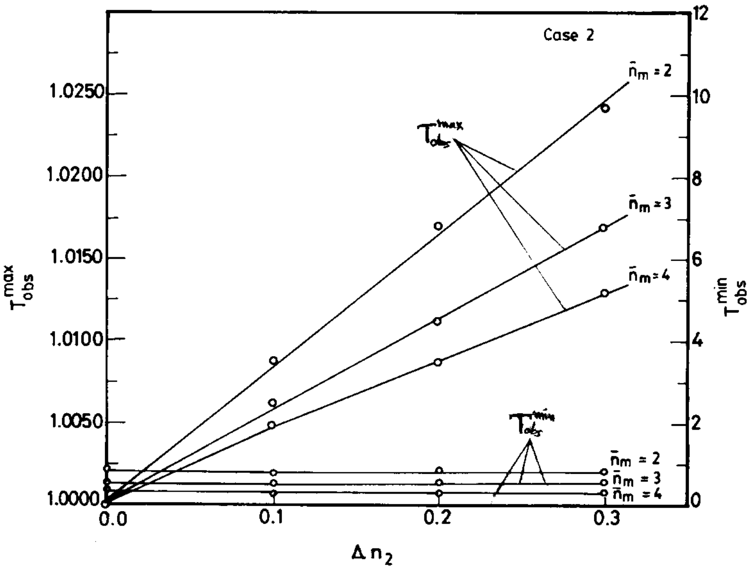


**Fig. (2)** Shows the effect of inhomogeneity of the film on the value of $T^{max}_{obs}$, $T^{\min}_{obs}$ in case 2.

## Case 3:

### $T_{f.coh}$ and $T_{s,coh}$ for inhomogenous films :

In this case $T^{\max}_{f,coh}$ , $T^{\min}_{f,coh}$ as in the previous case.But to obtain $T^{\max}_{s,coh}$ we must obtain $|r_f'|^2$ as a function of $n_f$ and $n_b$. So considering $\beta_1 = 2m\pi$ where m =0,1,2,3,...,then by substituting in equation (6) by $n_f$ and $n_b$ instead of $n_2$ we have

$$|r_f'|^2 = \frac{(n_1 n_f - n_b)^2}{(n_1 n_f + n_b)^2}$$

Then from equation (5) we have

$$T_{s,coh}^{max} = \frac{(n_1 n_f + n_b)^2}{n_1 (n_f + n_b)^2} \quad (23)$$

Accordingly

$$T_{sam}^{max} = \frac{4 n_f n_b}{(n_f + n_b)^2} \quad (24)$$

$$T_{obs}^{max} = \frac{4 n_f n_b}{(n_f + n_b)^2} \quad (25)$$

Also to obtain the minimum transmittance in this case we have to obtain $|r_f'|^2$ corresponding to $\beta_2 = (2m+1)\pi$.So by replacing $n_f$ , $n_b$ instead of $n_2$ in equation (6) we have

$$|r_f'|^2 = \frac{(n_b n_f - n_S)^2}{(n_b n_f + n_S)^2}$$

Then from equation (5) we have

$$T_{s,coh}^{min} = \frac{(n_f n_b + n_S)^2}{n_1 (n_f n_b + 1)^2} \quad (26)$$

Hence we can obtain $T_{sam}^{min}$ by using equation (19) and (26) as follows

$$T_{sam}^{min} = T_{f,coh}^{min} \cdot T_{s,coh}^{min} = \frac{4 n_f n_b}{(n_f n_b - 1)^2} \quad (27)$$

Then using equation (10) we have

$$T_{obs}^{min} = \frac{n_f n_b (n_w^2 + 1)^2}{(n_f n_b + 1)^2 n_w^2} \quad (28)$$

The calculations of $T_{obs}^{max}$ with $\Delta n$ for different values of $n_m$ show that $T_{obs}^{max}$ varies slightly with $\Delta n$. The result of calculations of $T_{obs}^{max}$ , $T_{obs}^{min}$ against

$\Delta n$ for different values of $n_m$ is illustrated in tables (3a,3b ,3a',3b'). Also the variation of the quantity ($T_{obs}^{max}$ -$T_{obs}^{min}$) versus $n_m$ is illustrated table (3c).

## Conclusions

1- The condition $T_{obs}^{max} > 1$ i.e $T_{max}$ higher than $T_w = 0.923$ for $n_2 = 1.5$ can be use d to distinguish the homogeneous and inhomogeneous films. Also the data illustrated in Figure (2) can be used to determined the degree of inhomogeneity $\Delta n$.

2- The increase of the quantity ($T_{obs}^{max}$ -$T_{obs}^{min}$) for weakly absorbing films is more pronounced in the case 1 and case 3, this can be explained by the occurence of interference in the substrate {see Tables (1)& (3)}.

3- The data illustrated in Figure (2) can be used to determine $n_m$ for inhomogeneous films.

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**Table (1). Variation of $T_{min}$ & $T_{obs}^{min}$ and the quantity $(T_{obs}^{max} - T_{obs}^{min})$ versus $n_2$ in case 1.**

| $n_2$ | 4 | 3.4 | 2 | 1.6 |
|---|---|---|---|---|
| $T_{min}$ | 0.221 | 0.293 | 0.640 | 0.808 |
| $T_{obs}^{min}$ | 0.260 | 0.344 | 0.751 | 0.948 |
| $(T_{obs}^{max} - T_{obs}^{min})$ | 0.740 | 0.656 | 0.249 | 0.051 |

**Table (2a). Variation of the quantity $(T_{obs}^{max} - T_{obs}^{min})$ versus $n_m$ for $\Delta n = 0.3$ in case 2.**

| $n_m$ | **4** | **3.4** | **2** | **1.6** |
|---|---|---|---|---|
| $(T_{obs}^{max} - T_{obs}^{min})$ | 0.680 | 0.581 | 0.190 | 0.054 |

**Table (2b). Variation of the quantity $(T_{obs}^{max} - T_{obs}^{min})$ versus $\Delta n$ for $n_m = 2$ in case 2.**

| **$\Delta$n** | **0.3** | **0.2** | **0.1** | **0.0** |
|---|---|---|---|---|
| $(T_{obs}^{max} - T_{obs}^{min})$ | 0.190 | 0.184 | 0.176 | 0.168 |

**Table (3a). Variation of $T_{min}$ and $T^{min}_{obs}$ versus $\Delta n$ as $n_m$ =2 in case 3.**

| $\Delta n$ | $n_m$ | $T_{min}$ | $T^{min}_{obs}$ |
|---|---|---|---|
| 0.3 | 2 | 0.644 | 0.754 |
| 0.2 | 2 | 0.643 | 0.753 |
| 0.1 | 2 | 0.642 | 0.752 |
| 0.0 | 2 | 0.640 | 0.751 |

**Table (3b). Variation of $T_{max}$ and $T^{max}_{obs}$ versus $\Delta n$ as $n_m$ =3.4 in case 3.**

| $\Delta n$ | $n_m$ | $T_{max}$ | $T^{max}_{obs}$ |
|---|---|---|---|
| 0.3 | 3.4 | 0.9979 | 0.9979 |
| 0.2 | 3.4 | 0.9991 | 0.9991 |
| 0.1 | 3.4 | 0.9998 | 0.9998 |
| 0.0 | 3.4 | 1 | 1 |

**Table (3b'). Variation of $T_{min}$ and $T^{min}_{obs}$ versus $\Delta n$ as $n_m$ =3.4 in case 3**

| $\Delta n$ | $n_m$ | $T_{min}$ | $T^{min}_{obs}$ |
|---|---|---|---|
| 0.3 | 3.4 | 0.294 | 0.345 |
| 0.2 | 3.4 | 0.293 | 0.344 |
| 0.1 | 3.4 | 0.292 | 0.343 |
| 0.0 | 3.4 | 0.291 | 0.34 |

**Table (3c'). Variation of the quantity ($T^{max}_{obs}$ - $T^{min}_{obs}$) versus $n_m$ for $\Delta n$ = 0.3 in case 3.**

| $n_m$ | 4 | 3.4 | 2 | 1.6 |
|---|---|---|---|---|
| ($T^{max}_{obs}$ - $T^{min}_{obs}$) | 0.738 | 0.653 | 0.240 | 0.039 |